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Intertemporal land allocation decision under uncertainty and hyperbolic discounting

Qi Wang

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Qi Wang

Supervisor: Luca Di Corato, Swedish University of Agricultural Sciences, Department of Economics

Examiner: Ing-Marie Gren, Swedish University of Agricultural Sciences, Department of Economics

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Abstract

Land is of prime importance for all the activities of human beings, and how to efficiently use the limited land resource has been a crucial issue ever since the earliest times of human society. Uncertainty and irreversibility are important issues related to land use change, and it has been shown by Mäler and Fisher (2005, p590-p592) that the replacement of stochastic variable by its expected value could result in inappropriate land use decisions.

Based on the two-period framework of Mäler and Fisher (2005, p590-p592), this study introduces two kinds of discounting schemes, standard exponential discounting and hyperbolic discounting, to their analysis. Hyperbolic discounting is demonstrated by many researches and it can decently illustrate individuals' time-inconsistent preferences toward future payoffs. The decision rules about land use under both exponential discounting and hyperbolic discounting are illustrated and compared in this thesis.

The results have shown that, under hyperbolic discounting, when an immediate reward is generated by converting the land for development, the land will be converted earlier than under standard exponential discounting. In contrast, when an immediate cost is entailed to land conversion, the land will be converted later under hyperbolic discounting.

Abbreviations

DM	Decision Maker
DU	Discounted Utility
FAO	Food and Agricultural Organization
UN	United Nations
WWF	World Wide Fund for Nature

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1 Introduction

Land, as one of the most important natural resources, plays a crucial role in the prosperity and development of human society. As a necessity, land is required for almost all kinds of activities of human beings. For instance, land is a necessary factor for agricultural production; landscape in terms of forest preserves biodiversity; human settlement and industrial production also need land as a basis for these activities (Anthony Young, 1998).

Land based agricultural production, such as growing crops, grazing of livestock, vegetable and fruit plantation, not only provides job opportunities for the local residents, but also contributes to the vast majority of human food supplies. According to a study from the Food and Agriculture Organization of the United Nations (FAO), during 1990-92 to 2010-12, the total number of people that suffer from hunger has been reduced by 132 million, which is around 6.1 % of the total population all over the world (FAO report, 2012). The growth of agricultural production, particularly the boost in crop yields, is the main reason for the decline in hunger in the past decades.

Another important purpose for the use of land is for forestry. Forests not only provide a diverse range of resources, like fuel woods and timber for human beings, but also function as habitats for most terrestrial organisms. It is reported that 90% of the world's terrestrial biodiversity, both animal and plant species, is contained by the forests (Living Planet Report 2010, World Wide Fund for Nature (WWF)). In addition, forests help to assimilate carbon dioxide, regulate the hydrological cycles, purify water, reduce soil erosion, and mitigate natural hazards such as landslides.

Land used for human settlements, such as houses, roads, parks and factories, occupies a large proportion of the whole land as well. With more people migrating to and settling in cities, the demand for urban land used for residential construction, public institutions like schools and hospitals, parks and urban green land for recreation and transportation will increase. All forms of human settlements depend on the use of land for these activities (Anthony Young, 1998).

In addition to these main purposes for land use such as food production, biodiversity conservation and urban development, land also plays an important role in terms of other functions like disposal of waste from human settlements, storage of water and mineral resources, and preserving both natural and historical sites (Anthony Young, 1998).

Land is of essential importance for human beings. Given the limited amount of land that is available all over the world and the ever-increasing population which puts pressure on the scarcity of land, special attention should be paid to the rational and appropriate use of land.

1.1 Problem background

Ever since the earliest times, changing the use of land has been an important way to fulfil human beings' demand for food and other necessities of life. Dating back to hundreds of years ago in the developed countries, and even recently in the tropical areas such as Brazil, Indonesia and Malaysia, a large amount of natural forests has been clear-cut and converted to agricultural land. It is the cultivation of land that has solved problems such as food shortages and demand for fuel woods. Another example could be the recent conversion from rural land to urban construction. With this trend towards urbanization in the developing countries, extensive expansion of cities has involved a large amount of land conversion especially in peri-urban areas. There are four issues closely related to this change in land use: competitiveness, irreversibility, uncertainty and discounting.

First of all, in most cases, there is a competition for land among different uses. That is, the same plot of land can be used only for one purpose during a certain period. Farming land, for instance, can be converted for a planned new highway. However, growing crops is no longer possible on the same plot of land any more. The clearance of a natural forest can be used for agricultural cultivation; however, the forest does not exist any longer. Therefore, when the land use is changed for one purpose, simultaneously, the possibility to use the land for another purpose is reduced.

Secondly, the land use change, or equivalently, the conversion of land, could be irreversible. In fact, the process of changing the use of land from its original state to human settlement, agricultural and industrial production has already resulted in a large amount of loss in biodiversity and other forms of environmental deterioration. The economic benefits of biodiversity could be potentially huge, and the costs of biodiversity loss and environmental deterioration could be the degradation or even collapse of nature's ecosystem. This in turn will threaten the well-being of humans, who closely rely on various kinds of services provided by the ecosystem. Therefore, once the land is converted, it will be rather costly and even environmentally impossible to reverse.

Thirdly, the changing of land use is usually risky. That is, when future outcomes for the land use are not known with certainty, possible states or outcomes of future land use should be taken into account for the optimal decision about the land use. Take the case of farming land for an example. Assume that the farming land is currently used for the purpose of growing wheat and the landowner is planning to convert this land for vegetable plantation. However, due to uncertain demand, the market price of vegetables could drastically decline or increase. And then, the future revenues from this new use of land are uncertain. Clearly, the future cannot be known with certainty. However, one thing of great importance is to take into account all the possible future outcomes when making decisions about land use changes.

Last but not least, as long as the land will be used for a certain purpose for several periods of time, an appropriate discount rate should be chosen to facilitate the Decision Maker (DM) to compare the different uses of land. In economics, discounting is used to compare an individual's preference of rewards at different points in time. Generally, in consideration of two similar rewards, individuals have a tendency to prefer the earlier reward than the later. Standard exponential discounting, which was introduced by Samuelson (1937), is traditionally used to illustrate this tendency. However, recent studies (for instance, Loewenstein and Thaler (1989), Loewenstein and Prelec (1992)) have shown a wide range of anomalies that challenged the core assumption of exponential discounting, i.e. time consistency. A new hypothesis, hyperbolic discounting, which is a revised mathematical model based on exponential discounting, has been observed and demonstrated by many studies (Frederick et al. (2002), Matthew J. Salois and Moss (2011)).

All in all, competitiveness between different land uses, irreversible conversion of land, uncertain future outcomes from land use change and adopting the right discount rate are important issues related to land use change. The optimal land use decision should take into account all four of these aspects.

1.2 Problem

With respect to issues about irreversibility and uncertainty for the land use change, more specifically in the context of commercial development of a preserved natural area, it was initially discussed by Arrow and Fisher (1974), and later set out by Mäler and Fisher (2005). They highlighted the irreversible effect of destroying a natural area, emphasized the intertemporal resolution of uncertainty, and concentrated on the intertemporal perspective of a decision about the development of the land. It was shown that, with the prospect of complete information which would resolve the uncertainty in the future, decision for current commercial development would be less likely to occur. The intuition behind the result was apparent: the prospect of complete information would motivate the DM to stay flexible in the future, and then making the best decision by taking advantage of the forthcoming information. The intertemporal resolution of uncertainty would also prevent the adoption of an irreversible decision now, i.e. commercial development, which would limit the DM's options in the future.

However, instead of choosing an appropriate discount rate, future benefits were treated as present values for their analysis (Arrow and Fisher 1974, Mäler and Fisher 2005). In this study, a discounting function will be introduced to the analysis proposed by Mäler and Fisher (2005). The study will compare the optimal land use decisions under different discounting schemes, standard exponential discounting and hyperbolic discounting.

On the one hand, comparing to the standard exponential discounting, which is characterized by a constant discount rate, hyperbolic discounting implies a declining discount rate over time. The DM under hyperbolic discounting tends to grab instant benefits or postpone the corresponding cost at the moment. That is, the hyperbolic discounting is more present-biased than the exponential discounting and cares more about present outcomes than future outcomes. On the other hand, the option of behaving optimally by delaying the decision about the development of the land should also be considered. The reason is that with the prospect of available information, the uncertainty will be reduced by simply waiting. As long as the land is maintained in an undeveloped situation, it is always available for the DM to use further information and make the optimal decision in the future. On the contrary, once the land is developed, the DM will be locked into development, and any future information does not contain economic value any more.

Given the prospect of behaving optimally by delaying development and the desire to obtain instantaneous benefits from present development, which is characterized by hyperbolic discounting, the DM will engage in an intertemporal tussle as to the development decision. The effect of hyperbolic discounting pushes the DM to take up the immediate payoffs by developing the land, while the option of postponement encourages the DM to wait patiently to see what will happen in the future. Therefore, in order to investigate the net effect of hyperbolic discounting and the option of postponement, in this thesis, a two-period framework, which consists of a first period and a single second period, will be constructed to explore the decision rules and results under hyperbolic discounting.

1.3 Aim

The aim of this study is to analyse the optimal land use decision under uncertainty and hyperbolic discounting. A two-period framework will be constructed to illustrate the effect of hyperbolic discounting:

In this two-period framework, the DM has the flexibility to make a land use change decision at the beginning of the first period or postpone the decision to the beginning of the second period. Decision rules and results under the standard exponential discounting and hyperbolic discounting will be analyzed. This two-period framework aims to figure out the following three aspects:

1) Comparison of decision rules under exponential and hyperbolic discounting;
2) Interpretation of the economical insights of the land use decisions under exponential and hyperbolic discounting; and
3) Sensitivity analysis of the corresponding parameters under hyperbolic discounting.

1.4 Outline of the study

As illustrated in Figure 1, this thesis includes six sections.

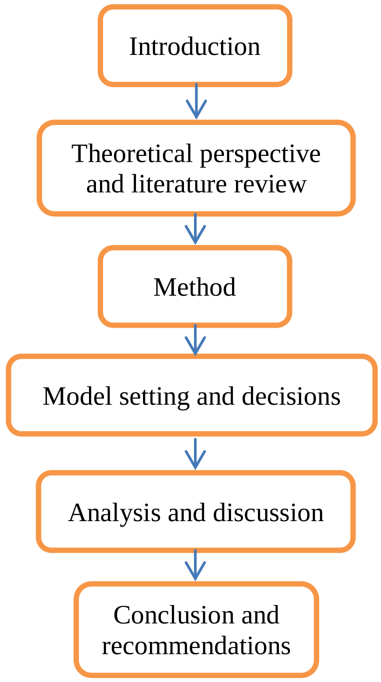

Figure 1. Illustration of the outline of the study

Chapter 1 gives an introduction about problem background, research questions and objectives of this study. A brief outline of this thesis is present in this section as well.

Chapter 2 provides the theory underpinning of this thesis. Theory about option value, quasi-option value in the context of irreversible development of land and uncertainty, theory about exponential discounting, time-inconsistent preferences and hyperbolic discounting are covered in this chapter.

Chapter 3 presents the research method used in this study, unit of study and delimitation are also highlighted in this chapter.

The model used for analysing optimal land use decision under uncertainty and hyperbolic discounting is provided in Chapter 4. Besides, optimal decisions about land use under different discounting schemes, exponential discounting and hyperbolic discounting, are also discussed and compared in Chapter 4.

Chapter 5 focuses on the analysis and discussion about the optimal decision about land use. The sensitivity analyses of corresponding variables, such as the discount rate and the probability are illustrated.

The last chapter presents the conclusions of this dissertation. Recommendations and suggestions for future research in this field are also provided in this chapter.

2 Theoretical perspective and literature review

The purpose of this chapter is to provide theoretical underpinnings for this study. This chapter reviews the theory of option value in the context of environmental economics. Literatures about intertemporal choice, exponential discounting, time-inconsistent preferences and hyperbolic discounting will be covered as well. This chapter concludes with the implication for this study, where the importance of this research is highlighted.

2.1 Option value in the context of environmental economics

The concept of option value was first discussed by Weisbrod (1964) in a context of uncertain demand for a national park, which might be closed. It was mentioned that if it was closed, no amenity services would be provided for any visitors. He argued that visitors here contained both current visitors and future potential visitors to the park. Therefore, in order to measure the benefit of keeping the park open, it was not appropriate to use only compensating consumer surplus to current visitors. The benefit to future potential visitors should be included to the benefit of preserving the park as well. According to Weisbrod, these potential visitors would pay certain amount of money to keep the option of visiting the park in the future, and the value attached to this future availability was denoted "*option value*".

As mentioned by Krutilla (1967, p780), "*option demand was characterized as a willingness to pay for retaining an option to use an area or facility that would be difficult or impossible to replace and for which no close substitute was available. Moreover, such a demand might exist even though there was no current intention to use the area or facility in question and this option might never be exercised*". He concluded that, an "*option demand*" or "*option value*" for preservation should be considered for an efficient allocation of resources.

Cicchetti and Freeman (1971) have shown that, the option value defined by Weisbrod (1964) would always be positive for a risk-averse individual. They concluded that the option value, which could be referred to as an extra benefit, was actually similar to a risk aversion premium. In their analysis, it was pointed out that only the expected value of individual's compensating surplus would underestimate the preservation benefit of keeping the natural park open. The underlying reason was that risk-averse individuals would like to pay a premium to avoid the risk of losing the possibility to visit the park.

The brief idea of option value was actually that individuals would be willing to pay more money than the expected consumers' surplus in order to secure the option of visiting the park in the future. However, some controversy was triggered for the precise definition and measure of this option value. Schmalensee (1972) argued that option value might be either positive or negative and the expected consumer surplus should be the best available estimation of option prices in the presence of uncertainty and risk aversion. He also pointed out that the benefit would be either underestimated or overestimated with this approximation, but no better estimation could be available. Bohm (1975) did agree with Schmalensee's result about the sign of option values in the context of uncertain future preferences and risk aversion. However, he also pointed out that the estimation for expected consumers' surplus was as difficult as the estimation for option prices. The research result of Graham (1981) has shown that it was better to use option price as a measure of benefit in the case of similar individuals and collective risk, on the contrary, expected consumer surplus was a better measure of benefit in case of similar individuals and personal risks.

Another challenge to the option value analysed by Cicchetti and Freeman (1971) was the research provided by Arrow and Lind (1970). Arrow and Lind (1970) have shown that when the risk of an investment was undertaken by an increasing amount of individuals, the aggregate risk premiums of all individuals tended to be zero. Thus, the most important criterion for evaluating a project was the expected return to the investment.

Arrow and Fisher (1974) focused on the intertemporal perspective of decision making about the development of a preserved natural area. This natural area also provided amenity services under the purpose of preservation. Given the assumptions that this commercial development would result in a perpetuity loss from preservation, and forthcoming information would reduce the uncertainty about future benefits from both development and preservation, less amount of land should be converted for development in the first period. The core task of this research was checking the effect of introducing stochastic costs and benefits instead of using expected values of these random variables. They argued that even where it was not appropriate to assume risk aversion, the irreversibility of a development could lead to an effect similar with risk aversion.

Henry (1974a, p 1006) defined an "*irreversible decision*" in this way, "*a decision is considered irreversible if it significantly reduces for a long time the variety of choices that would be possible in the future*". He discussed the effect of irreversibility in the context of a proposed project about building a new highway around Paris which might destroy public parks and historical sites. It was mentioned in his discussion that the uncertain benefits or costs involved in a project were usually displaced by their expected values. That is, a riskless problem was used as a substitution for the original stochastic problem. He pointed out that an irreversible decision would always be favoured by this substitution, and the size of "*irreversibility effect*" played an important role in the process of decision making.

Henry (1974b) pointed out that Cicchetti and Freeman (1971) and Arrow and Fisher (1974) had shown two kinds of interpretations for the concept of "*option value*". In his research, Henry focused on the effect of complete information in the decision-making process. The result showed that the preservation of a historical site or natural area would always be favoured, even little prospect of complete information would be available. In other words, given a mere possibility of obtaining complete information and the irreversibility of development, a positive option value would be generated by preserving the historical site or natural area. This was just the "*irreversibility effect*" which was mentioned by Henry (1974a).

As mentioned above, two interpretations of Weisbrod's option value have arisen (Hanemann 1989). The first, presented by Cicchetti and Freeman (1971), and further refined by Schmalensee (1972), Bohm (1975) and Graham (1981), have shown that option value was similar to a risk premium that many consumers would be willing to pay to preserve the park for future visit. They argued that this option value should be taken into account in social benefit-cost analysis for the project of closing the natural park. The second interpretation, presented by Arrow and Fisher (1974), and Henry (1974a, 1974b), focused on the irreversibility of closing the park and the intertemporal decision with prospect of complete information about future benefits. The later interpretation of option value was also referred to as Arrow-Fisher-henry (AFH) option value, or quasi-option value in the literature.

Freeman (1984) pointed out that the quasi-option value was in fact a neutral concept. Neither preservation nor development would be preferred in the presence of a quasi-option value. The existence of quasi-option value and its sign largely relied on the nature of uncertainty, the possibility of obtaining information and structure of the decision problem. With respect to the challenge from Freeman (1984) and Miller and Lad (1984), who also argued that development decisions involving a quasi-option were not necessarily more conservative than decisions without the quasi-option. Fisher and Hanemann (1987) claimed that the quasi-option value would never be negative, but the net benefit from preservation could be either positive or negative. Besides, it was proved by Fisher and Hanemann, when the uncertainty mainly came from the benefits of development, decision would favour for development; when the uncertainty was primarily about the benefits of preservation, decision of postponing irreversible development would be favoured.

Hanemann (1989) claimed that the irreversibility of development has led to such a situation: in case of preservation in the initial period, it was always possible to make the best decision later according to the subsequent information about future benefits of development and preservation; in case of development in the initial period, it was not available to change the initial decision and the further information available was actually meaningless. He concluded that the AFH option value, or the quasi-option value, was the value of flexible decision in the future.

Recently, a bunch of research about the irreversible development of an environmental resource have relied on the real option approach proposed by Dixit and Pindyck (1994). With the arisen concern about the relationship between this real option and the option value developed in the context of environmental economics, Fisher (2000) proved that they were actually equivalent.

2.2 Intertemporal choice and exponential discounting

Intertemporal choice is the study of how individuals evaluate costs and benefits at different points in time and make decisions that also influence their choices in the future (Berns et al. 2007). The discounted utility (DU) model, which was introduced by Samuelson (1937), is used as a standard economic method to compare trade-offs between costs and benefits at different points in time. As stated by Frederick et al. (2002), the most important reasons why DU model is so popular for analysing intertemporal decisions is its simplicity and similarity with the compound interest formula. That is, the DU model assumes that individuals evaluate the costs and benefits of a decision in a similar way that financial market evaluates monetary gains and losses. Future costs and benefits are all exponentially discounted in accordance with how delayed they are over a time horizon. The most important assumption of the DU model is that intertemporal preferences of an individual can be characterize by a single, constant discount rate.

Given a discount function $f(\tau)$, the instantaneous discount rate at time τ is defined as $-\frac{f'(\tau)}{f(\tau)}$ (Laibson 1997). In particular, if the discount function takes the form $D_t = \left(\frac{1}{1+\rho}\right)^t$. The instantaneous discount rate at time $t\ is\ \rho_t = \frac{D_t - D_{t+1}}{D_{t+1}} = \rho$. That is, ρ is the instantaneous discount rate for all periods in a DU model. Constant discounting describes how an individual evaluate payoffs at different points in time. It means that putting off or bringing forward two payoffs should not change individual’s preference toward the payoffs. Take the opportunity of an investment as an example, if it is optimal to make the investment now, it should also be optimal to make the investment in any time in the future. The assumption of constant discounting allows a person’s time preference to be compressed into a single discount rate. If constant discounting does not hold, an entire discount function should be specified in order to characterize an individual’s time preferences.

As shown in Koopmans' (1960) axiomatic derivation of the DU model (with exponential discounting), the postulate of stationarity was the reason for a constant discount rate. Even though neither Samuelson nor Koopmans recommended the DU model as a normative model for intertemporal choice, the simple DU model was widely adopted by economists. It was used as the framework to analyse intertemporal decisions. According to the DU model, the only difference between intertemporal choice and other types of choices is that, some consequences under intertemporal choice are delayed, and hence they must be discounted. Therefore, many researches in the field of intertemporal choices have focused on choosing an appropriate discount rate.

2.3 Time-inconsistent preference and hyperbolic discounting

Strotz (1956) first pointed out that individuals tended to be more impatient when they compared short-run payoffs than when they compared long-run payoffs. He suggested that, at different points of time, different discount rates should be employed to illustrate individuals' time-inconsistent preferences. Strotz did not propose any specific mathematical form for the phenomenon of time-inconsistent preference, however, he did draw attention to the case of declining discount rates. Later studies on time preferences have actually supported his result. Numerous experiments with animals, notably pigeons, have shown that animals discounted the future rewards in a non-exponential manner. For example, the research of Rachlin and Green (1972) indicated that, when provided with a choice between a small immediate reward (2-sec exposure to grain) and a large reward (f exposure to grain) four seconds later, pigeons always preferred the small, immediate reward. However, when this choice was postponed T seconds, pigeons would make choices according to the delayed time T. When T was short, pigeons chose the small immediate reward. When T was long, pigeons only chose the large delayed reward. Ainslie (1975) suggested that impulsiveness appeared to be the reason of employing hyperbolic curves to illustrate the decline of rewards over the course of time.

Researches about humans also suggested the same time-inconsistent preferences. An example given by O'Donoghue and Rabin (1999) has showed that: when provided with a choice between doing seven hours of unpleasant task on the first of April and eight hours on the 15^{th} of April, individuals would always prefer seven hours on the first of April if they were asked on the first of February. However, when the first of April came, given the same choice, most of individuals would prefer to put off the unpleasant task to the 15^{th} of April. As mentioned by Berns et al. (2007), most humans did concern about, or at least be able to take into account costs and benefits at different points in time. On the contrary, research by Stevens et al. (2005) has shown that the closest evolutionary relatives of humankind, cotton-top tamarin monkeys, would always choose an immediate food reward rather than waiting eight seconds for a triple reward.

Time-inconsistent preference, that is, individuals are more impatient in the short run than in the long run, is often referred to as present biased preferences (O'Donoghue and Rabin 1999). This preference is well described by a hyperbolic discounting function. Hyperbolic discounting functions decline at a fast rate in the short run and decrease at a slow rate in the long run. Therefore, a hyperbolic discounter is more impatient when compare short-run payoffs than when comparing long-run payoffs.

The first formal model about hyperbolic discounting was introduced by Chung and Herrnstein (1967). According to their experimental research about pigeons, the mathematical form of $D(t) = 1/t$ was used as the discounting function to describe the behaviour of pigeons. The general hyperbolic discounting function was developed by Loewenstein and Prelec (1992). They adopted a generalized hyperbola,

$$\mathrm{D}(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}}, with\ \alpha, \beta > 0, \tag{2.3.1}$$

where the level of deviation from exponential discounting was determined by the parameter α. In the limiting case, when α approached to zero, this hyperbolic discount function became $D(t) = e^{-\beta t}$. And it is the same as exponential discounting.

Phelps and Pollak (1968) originally took a discount function of the form

$$D(t) = \begin{cases} 1, & if\ t = 0 \\ \beta\delta^t, & if\ t > 0 \end{cases}, \text{ where } 0 < \beta < 1\ and\ 0 < \delta < 1, \quad (2.3.2)$$

to discuss intergenerational altruism. This discount function is often referred to as quasi-hyperbolic discounting. In a quasi-hyperbolic discount function, the discount factor δ reflected individual's time preference. While the constant factor β, applied equally to each period t, implies that the discount factor between current period and the next is lower than the discount factor in later periods. Therefore, β could be interpreted as a measure of individuals' present bias. When $\beta = 1$, the quasi-hyperbolic discount function is simply the exponential one. Quasi-hyperbolic discount function, with a discrete time function structure $\{1, \beta\delta, \beta\delta^2, \ldots\}$, is often referred to as (β, δ) model. Comparing with the traditional DU model, the utilities in a time horizon $(0,1,2, \ldots T)$ are discounted by $\{1, \beta\delta, \beta\delta^2, \ldots, \beta\delta^T\}$ in the case of a (β, δ) model (Wilkinson and Klaes 2012, p111).

Laibson (1997) adopted this (β, δ) model to approximate the qualitative property of the general hyperbolic discounting function, as shown in equation (2.3.2). It is clear that (β, δ) model maintains most of the analytical tractability of the traditional exponential model. Specifically, at period one, the discount factor is $\beta\delta$; after period one, the discount factor between two periods is just δ, the same as exponential discounting.

This (β, δ) model was adopted by O'Donoghue and Rabin (1999) to elegantly simplify the present-biased preferences. The effectiveness of (β, δ) model relies on the assumption of a higher instantaneous discount rate between current and the next period (indicating the time-inconsistency), but a constant instantaneous discount rate between any two future period [1]. A common strategy used to describe time-inconsistent preferences is to model each individual at different time points as separate "*selves*". Current "*self*" behaves optimally and maximizes his/her life-time utility, while all the future "*selves*" are assumed to behave optimally and able to maximize his/her corresponding utilities in the future. As mentioned by Pollak (1968), there were two extreme assumptions about one individual's belief about his/her future selves: A person could hold a naïve belief, which implied that the person simply believed future "*selves*" would have the same preferences just as the current "*self*". That is, the person did not realize the problem that, with the approaching of time, future "*selves*" could make different choices, which would not be preferred by the current "*self*". A person could hold

[1] For the (β, δ) model, the instantaneous discount rate between current and the next period is $\frac{1-\beta\delta}{\beta\delta}$, while the instantaneous discount rate between any two future periods is $\frac{1-\delta}{\delta}$. Clearly, we have $\frac{1-\beta\delta}{\beta\delta} > \frac{1-\delta}{\delta}$.

a sophisticated belief, which implied that the person anticipated the preference reversal of his/ her future "*selves*".

Whether individuals are sophisticated or naïve is still controversial between economists. Akerlof (1991) assumed naïve beliefs in his analysis about procrastination and obedience. Nevertheless, most economists assumed sophisticated beliefs when they modelled time-inconsistency preferences. The reason for sophisticated beliefs is that people have "*rational expectations*" about future behaviours. O'Donoghue and Rabin (1999) argued that both sophistication and naivety were possible when individuals make anticipations about their future preferences. It was mentioned that, the use of self-commitment, like alcohol club, and fat farms provided evidence for sophisticated behaviours. As explained by him, only sophisticated agents could anticipate preference reversals of their future "*selves*" and therefore made a commitment to control their future "*selves*". However, it indeed was shown that people did underestimate the level of deviation from their current preferences. The example given by O'Donoghue and Rabin (1999) was that individuals may frequently lose the "*willpower*" to resist the immediate temptations, whilst optimistically anticipated that they would regain the "*willpower*" to resist temptations later.

Another research by O'Donoghue and Rabin (2001) allowed the person to be partially naïve, that is, the problem of future preference reversal was perceived by the person, however, the magnitude of it was underestimated. In order to model the partially naïve behaviour, a parameter $\hat{\beta}$ was introduced to measure an individual's belief about immediate gratification. In case of sophistication, a person was aware of the future preference reversal, and anticipated $\hat{\beta} = \beta$. In case of naivety, a person would not realize future preference reversal, and anticipated $\hat{\beta} = 1$. In case of partial naivety, a person anticipated $\hat{\beta} \in (\beta, 1)$. Results obtained by them have shown that any level of naivety could lead to results that were different from the prediction under complete sophistication.

In order to illustrate the effect of partial naive belief, various empirical evidences was discussed by DellaVigna (2009). Studies about choosing between a monthly contract and pay-per-visit contract for a gym, effect of deadlines on homework completion and the setting of deadlines, credit card usage with different interest rates, default effects in retirement saving were discussed. It was shown that, for a partial naive agent, the consumption of investment good was overestimated, and the consumption of leisure good was underestimated. Characteristics about investment good were that efforts were required now and rewards would be delivered later. On the contrary, characteristics about consumption good were that an immediate reward was generated and a future cost would be involved.

2.4 Implication for this study

As highlighted by Arrow and Fisher (1974), Henry (1974a, 1974b), Fisher and Hanemann (1987), Hanemann (1989), Mäler and Fisher (2005), the replacement of a stochastic variable by its expected value could lead to inappropriate land use decision. Therefore, this thesis will focus on the case that the stochastic variable will not be replaced by its expected value.

This thesis targets to analyse the land conversion decision made by present-biased DM, whose preference is well illustrated by a quasi-hyperbolic discounting function, under uncertainty and irreversibility. In order to illustrate the instantaneous effect of hyperbolic discounting, a revised model, basing on Mäler and Fisher (2005)'s two-period framework, is constructed. Besides, the decision rules under standard exponential discounting, a time-consistent preference, is also illustrated as a benchmark. The extra flexibility and value generated by option to postpone the land conversion decision is also evaluated.

3 Method

Given the theoretical underpinnings provided in the previous chapter, this one presents the research methods applied in this thesis. It starts with two traditions about research in social science: quantitative research and qualitative research. Then, the choice of method for this study and the unit of analysis are presented. In the last section, delimitation is highlighted.

3.1 Research in social science

Traditionally, quantitative research and qualitative research have been regarded as two basic research paradigms in social sciences. The differences between these two research paradigms are based on distinct philosophical assumptions about the reality: the former assumes that the reality is what observed by people and the latter believes that the reality is what constructed by human mind (Colin Robson, 2011).

The first approach, quantitative research, emphasizes the importance of following the research methods used in natural sciences, such as physics, mathematics and chemistry, and claims that this is the scientific way to do research. The advocators of quantitative research insist that quantification of the obtained information and numerical analysis are of essential importance. In fact, quantitative research has been closely related to positivistic view of research, which insists that observation and experience are the direct sources for objective facts, and researchers have no influence on the phenomenon observed. However, criticisms of this positivistic view, like characteristics and values of the researchers will affect their observations, has advocated a post-positivistic approach. This post-positivistic view claims that, research is the process of exploring the universal laws, evidence is always not enough for researches and conclusions should be refined and examined given new observations. Recently, some followers of quantitative research go on with the old positivistic route, while others take into account these criticisms and go for post-positivistic research (Colin Robson, 2011).

The second one highlights that the object of social research is human beings, who have consciousness and behave according to their willingness. Human beings are totally distinct from the general research objects in natural science, which can not perceive and react to what is happening around. And therefore, a different research approach other than the quantitative route used in natural sciences should be taken into account.

It is not appropriate to say one research paradigm is better than the other. Both quantitative and qualitative approaches can be used to obtain certain research purposes. In fact, the choice of research paradigm largely depends on the content and purpose of the study. Recently, there is an increasing recognition about a mixed-methods approach, which combines the elements from both qualitative and quantitative research. And a multi-strategy design which takes the advantages from both qualitative and quantitative research is advocated by many practising researchers (Colin Robson, 2011).

3.2 Choice of method

3.2.1 A dynamic approach

This thesis focuses on the intertemporal aspect of decision about land use change. The traditional cost benefit analysis, which is a 'now or never' evaluation criterion, does not fit the research target of this thesis. In order to model the intertemporal decision about land use change, a dynamic approach will be used in this study.

Dynamic approach here has three crucial features. First, the decision about land use change form preservation to development is assumed irreversible. That is, once the land is developed, it is not possible to reallocate the land back for preservation. Second, it is flexible for the DM to make this irreversible decision now or later, i.e. the decision about land use change could be delayed. Third, more information about future payoffs will come and the uncertainty will be reduced.

Assume the net benefit from preservation and development now is $B_{pre\ 1}$ and $B_{dev\ 1}$, respectively; the net benefit from preservation and development later is $B_{pre\ 2}$ and $B_{dev\ 2}$, respectively. The overall benefit from preservation is denoted by *P* and the overall benefit from development is denoted by *D*. The purpose of this section is to develop a decision making model to illustrate the decision rules with respect to the land use change. It is implicitly assumed that the DM will choose the decision that could generate the highest net present benefits.

In the future, the DM could choose the best decision according to the complete information about net benefit from preservation and development $B_{pre\ 2}$ and $B_{dev\ 2}$. Denote B_2 as the benefit later, we have

$$B_2 = E\left[\max\left\{B_{pre\ 2,} B_{dev\ 2}\right\}\right]. \qquad (3.2.1)$$

If the DM goes for preservation now, the overall benefit will be

$$P = B_{pre\ 1} + E\left[\max\left\{B_{pre\ 2,} B_{dev\ 2}\right\}\right] * Discount\ Factor. \qquad (3.2.2)$$

If the DM goes for development now, later he/she will be locked in the development. The overall benefit will be

$$D = B_{dev\ 1} + E[B_{dev\ 2}] * Discount\ Factor. \qquad (3.2.3)$$

When $P > D$, the DM will choose to preserve the land now; when $P < D$, the DM will decide to develop the land now; and when $P = D$, the DM will be indifferent between preservation and development.

3.2.2 The choice of discounting functions

Exponential discounting, which is characterized by a constant discount rate, is the standard discounting function used to illustrate individual's intertemporal preferences. The discount structure of exponential discounting is

$$\{1, \delta, \delta^2, \dots\}. \tag{3.2.4}$$

Exponential discounting is analytically simple. It is often referred to as time-consistent discounting, since it implies that the preferences of individuals toward two outcomes will not change no matter when they will be asked. However, more evidences have shown that individual's preferences will change according to the passage of time. And they usually discount the short-run payoffs at a higher discount rate whilst discount the long-run payoffs at a lower discount rate. This present-biased preference could be well illustrated by a generalized hyperbola, which was first adopted by Loewenstein and Prelec (1992).

$$\mathrm{D}(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}}, with\ \alpha, \beta > 0. \tag{3.2.5}$$

Laibson (1997) adopted a quasi-hyperbolic discounting function, which was first proposed by Phelps and Pollak (1968), to approximate the qualitative property of the general hyperbolic discounting function (3.2.5).

$$D(t) = \begin{cases} 1, & if\ t = 0 \\ \beta\delta^t, & if\ t > 0 \end{cases}, \text{ where } 0 < \beta < 1\ and\ 0 < \delta < 1, \tag{3.2.6}$$

The discount structure of quasi-hyperbolic discounting function is

$$\{1, \beta\delta, \beta\delta^2, \dots\}. \tag{3.2.7}$$

It is clear that this discount structure $\{1, \beta\delta, \beta\delta^2, \dots\}$ maintains most of the analytical tractability of the exponential model, for which the discount structure is $\{1, \delta, \delta^2, \dots\}$. At period one, the discount factor for quasi-hyperbolic discounting is $\beta\delta$; after period one, the discount factor between two periods is just δ, which is the same as exponential discounting.

This thesis will use the quasi-hyperbolic discounting, as shown in (3.2.6) to analyse the influence of hyperbolic discounting for the optimal land use decision.

3.3 The unit of analysis

This thesis focuses on the study of a general land allocation decision between two competitive purposes, preservation and development. The factors affecting the decision making process of converting the land from preservation to development are studied.

In order to illustrate the decision rules and results under hyperbolic discounting, the analysis under exponential discounting is discussed for comparison. In addition, an approximation of hyperbolic discounting, quasi-hyperbolic discounting, was used in the model setting. Quasi-hyperbolic discounting not only keeps the qualitative property of the general hyperbolic discounting function, but also maintains most of the analytical tractability of the standard exponential discounting function.

3.4 Delimitations

This study is a theoretical research about land allocation decision between two possible destinations: preservation and development. There could be some challenges about the assumptions of irreversible development and intertemporal resolution of uncertainty. However, when the developed land can only be used for a certain purpose, and more information would be available with the passage of time, these assumptions seem rather reasonable. Therefore, in this sense, the analysis is meaningful in order to solve real-life issues.

This study is based on the two-period framework of Mäler and Fisher (2005), which is adequate to illustrate the main conclusions for irreversibility and intertemporal resolution of uncertainty. The simplicity of mathematics and effectiveness to show the main results after introducing hyperbolic discounting are the reasons for choosing this framework. Other theoretical framework, which is mainly about continuous time and stochastic processes, are not considered.

4 Model setting and decisions

Assume a risk neutral DM owns one unit of land and he/she is concerned with the best use of this land. The best decision is defined in such a way that the net present value (benefits minus costs) of this plot of land is maximized. For the sake of simplicity, two possible uses of this land are considered: preservation and development. In other words, the land can be preserved in an original undeveloped situation, or be converted for developing purposes. An example of this kind of problem could be the decision about the use of a farming land. On the one hand, this land can be preserved and used for the initial purpose of agricultural production; on the other hand, this land could be converted for residential construction as well.

Clearly, the decision about land use change between preservation and development could be put into effect at any point of time. However, the essential results can be decently illustrated by a discrete two-period model. For this two-period analysing framework, only a first period and a second period, which could also be referred to as now and future, are needed.

There are five core assumptions about this model: (1) this unit of land could either be preserved or developed as a whole in the sense that partial development of this unit of land is not considered in this thesis; (2) this unit of land is initially undeveloped and it is flexible for the DM to develop the land now or in the future. The development of land is assumed to be irreversible. In other words, once the land is developed, it is not possible to reallocate the land for other purposes; (3) a cost will be involved in the period which the development of land is undertaken; (4) benefits from preservation and development in the first period are assumed to be known; the cost related to development is certain as well; (5) Benefit of preservation and development in the second period is stochastic, but it is assumed that information about future benefit will be available and taken into account at the start of the second period.

4.1 Model setting

An irreversible decision about land development can be made in the first period or be postponed to the second period. Assume the net benefit from first period development is $B_1(d_1)$, where $d_1 \in \{0,1\}$ is the decision about the level of development in the first period. Therefore, net benefit from the first period preservation is represented by $B_1(0)$ and net benefit from the first period development is represented by $B_1(1)$. By assumption, $\mathrm{B}_1(0)$ and $\mathrm{B}_1(1)$ are known with certainty.

The net benefit from second period development is $B_2(d_1, d_2)$, where $d_1 + d_2 \in \{0,1\}$ and d_2 is the decision about the level of development in the second period. Because the development is irreversible, we have $\mathrm{d}_2 = 0$ in the case of $\mathrm{d}_1 = 1$; and $\mathrm{d}_2 \in \{0,1\}$ in the case of $\mathrm{d}_1 = 0$. By assumption, this net benefit from second period development is stochastic. Therefore, there will be two notations for the net benefit from second period development, $B_2(1{,}0)$ and $B_2(0{,}1)$. The numbers in parentheses are the levels of first-period and second period development, respectively. Because there will be cost in the period which the development is undertaken, the former must be higher than the latter. In other words, we must have $\mathrm{B}_2(1{,}0\ ) > B_2(0{,}1)$.

Suppose there are only two possible situations in the second period, state 1 and state 2. And the DM holds the belief that, state 1 will occur with probability π, and that the state 2 will occur with probability $1 - \pi$. When state 1 happens, net benefit from second period development, $B_2^1(0{,}1)$, is higher than the net benefit from preservation, i.e. $B_2^1(0{,}1) \geq B_2^1(0{,}0)$; And when state 2 occurs, net benefit from development, $B_2^2(1{,}0)$, is lower than the net benefit form preservation, i.e. $B_2^2(1{,}0) < B_2^2(0{,}0)$.

In what follows, exponential discounting and hyperbolic discounting will be taken into account, respectively.

4.1.1 Exponential discounting

We first consider the standard exponential discounting in our model. Notice that second period development decision, d_2 will be decided at the start of the second period when the information about whether no development or full development yields higher benefit is available. At the start of the first period, when d_1 must be decided, we only have the expected value of the maximal payoff for the second period decision.

Given the development decision d_1 chosen in the first period, thus, we have the expected value of the maximal net benefits in period 2

$$\hat{B}_2(d_2) = E\left[\max_{d_2, d_1 + d_2 \leq 1} B_2(d_1, d_2; \pi)\right]. \tag{4.1.1}$$

The first period decision should be consistent with the maximization of expected net payoffs over both periods. Let's define $\hat{V}(d_1)$ as the expected payoffs over both periods. We know that $d_1 = 0$ or $d_1 = 1$. Assume the exponential discounting factor is $\delta = \frac{1}{1+r}$, where r is the discount rate for the interval of one period.

In case of $d_1 = 1$, that is, it is chosen to develop the unit of land in the first period. Because of irreversibility, in the second period there is no choice but keeping the land being developed. We have, the expected payoffs over both periods,

$$\hat{V}(1) = B_1(1) + \hat{B}_2(0) * \delta = B_1(1) + E[B_2(1; 0; \pi)] * \delta. \tag{4.1.2}$$

In case of $d_1 = 0$, that is, it is chosen to postpone the development of the unit of land to the second period. In fact, when it comes to the second period, the information about whether preservation or development provides a higher benefit is available. However, at the start of first period, only the expectation of the maximum is available. We have, the expected payoffs over both periods,

$$\hat{V}(0) = B_1(0) + \hat{B}_2(d_2) * \delta = B_1(0) + E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] * \delta. \tag{4.1.3}$$

4.1.2 Quasi-hyperbolic discounting

Based on recent researches about intertemporal choice, it has shown that when individuals evaluate costs and benefits at different points if time, instead of discounting the payoffs in the future exponentially, individuals tend to discount the future payoffs hyperbolically. That is, they discount the short-run outcomes at a higher discount rate and discount the long-run outcomes at a lower discount rate. This present-biased preference of individuals could be well illustrated by a quasi-hyperbolic discounting function

$$D(t) = \begin{cases} 1 & , if\ t = 0, \\ \beta\delta^t & , if\ t > 0. \end{cases} \tag{4.1.4}$$

where t is the delayed time of future payoffs, $\beta \in [0,1]$ is the hyperbolic factor, and $\delta = \frac{1}{1+r}$, where r is the discount rate for the interval of one period, is just standard exponential discounting factor. When $\beta = 0$, the DM is an extreme hyperbolic discounter, who only cares about current outcomes, regardless of any consideration about future. When $\beta = 1$, the DM is a standard exponential discounter.

Similar with the case of exponential discounting, the first period decision should be consistent with the maximization of expected net payoffs over both periods. Let's define $\tilde{V}(d_1)$ as the expected payoffs over both periods. We know that $d_1 = 0$ or $d_1 = 1$. By adopting quasi-hyperbolic discounting rather than exponential discounting, we have:

In case of $d_1 = 1$, that is, it is chosen to develop the whole land in the first period. Because of irreversibility, in the second period we have no choice but keep the whole land being developed. We have, the expected payoff over both periods,

$$\tilde{V}(1) = B_1(1) + \hat{B}_2(0) * \beta\delta = B_1(1) + E[B_2(1,0; \pi)] * \beta\delta. \tag{4.1.5}$$

In case of $d_1 = 0$, that is, it is chosen to postpone the development of the whole land to the second period. In fact, when it comes to the second period, it is known that whether preservation or development provide the higher benefit. However, at the start of the first period, only the expectation of the maximum is available. We have, the expected payoff over both periods,

$$\tilde{V}(0) = B_1(0) + \hat{B}_2(d_2) * \beta\delta = B_1(0) + E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] * \beta\delta. \tag{4.1.6}$$

4.2 Decisions

4.2.1 Decision rules under exponential discounting

In order to get the decision rules for the first period, $\hat{\mathrm{d}}_1$, we need to compare $\widehat{\mathrm{V}}(0)$ and $\hat{V}(1)$, that is,

$$\widehat{\mathrm{V}}(0) - \widehat{\mathrm{V}}(1)$$
$$= \mathrm{B}_1(0) - \mathrm{B}_1(1) + \delta * \left\{\mathrm{E}\left[\max_{\mathrm{d}_2 \in \{0,1\}} \mathrm{B}_2(0, \mathrm{d}_2; \pi)\right] - \mathrm{E}[\mathrm{B}_2(1,0; \pi)]\right\}, \qquad (4.2.1)$$

and choose

$$\hat{d}_1 = \begin{cases} 0, & if\ \hat{V}(0) > \hat{V}(1), \\ 0\ or\ 1, & if\ \hat{V}(0) = \hat{V}(1), \\ 1, & if\ \hat{V}(0) < \hat{V}(1). \end{cases} \qquad (4.2.2)$$

Therefore, land is developed in the first period, when $\hat{V}(0) < \hat{V}(1)$; the decision of development will be postponed to the second period, when $\hat{V}(0) > \hat{V}(1)$; the DM will be indifferent between these two choices, when $\hat{V}(0) = \hat{V}(1)$.

4.2.2 Decision rules under quasi-hyperbolic discounting

In order to derive the decision rules for the first period, $\tilde{d}_1$, we need to compare $\tilde{V}(0)$ and $\tilde{V}(1)$, that is,

$$\tilde{V}(0) - \tilde{V}(1)$$
$$= B_1(0) - B_1(1) + \beta\delta * \left\{E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] - E[B_2(1,0; \pi)]\right\}, \qquad (4.2.3)$$

and choose

$$\tilde{d}_1 = \begin{cases} 0, & if\ \tilde{V}(0) > \tilde{V}(1), \\ 0\ or\ 1, & if\ \tilde{V}(0) = \tilde{V}(1), \\ 1, & if\ \tilde{V}(0) < \tilde{V}(1). \end{cases} \qquad (4.2.4)$$

Therefore, land is developed in the first period, when $\tilde{V}(0) < \tilde{V}(1)$; the decision of development will be postponed to the second period, when $\tilde{V}(0) > \tilde{V}(1)$; and the DM will be indifferent between these two choices, when $\tilde{V}(0) = \tilde{V}(1)$.

5 Analysis and discussion

5.1 Analysis

5.1.1 Analysis under exponential discounting

According to the decision rules illustrated by equation (4.2.2) in the section of 4.2.1, the level of development in the first period, $\hat{d}_1$, depends on the expected payoffs over both periods $\hat{V}(\hat{d}_1)$. $\hat{d}_1$ can either be 0 or 1. Therefore, the decision about first period development will be determined by the value of $\hat{V}(0)$ and $\hat{V}(1)$.

Consequently, there will be three different situations: $\hat{V}(0) > \hat{V}(1)$, $\hat{V}(0) < \hat{V}(1)$ and $\hat{V}(0) = \hat{V}(1)$.

<u>Case 1:</u> $\hat{V}(0) > \hat{V}(1)$ *and* $\hat{d}_1 = 0$. Under this situation, the land will not be converted in the first period. And it is optimal for the DM to wait and decide whether to convert the land or not in the second period. Therefore, we have

$$B_1(0) + \delta * \left\{ E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi) \right] \right\} > B_1(1) + \delta * E[B_2(1,0;\pi)]. \qquad (5.1.1)$$

For the sake of simplicity, define $A = B_1(1) - B_1(0)$. "A" is the difference between net benefits from development and preservation in the first period. Define $B = E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] - E[B_2(1,0;\pi)]$. "$B$" is the difference of net benefits in the second period by choosing preservation and development in the first period.

Rearrange the inequality (5.1.1), and substitute in *A* and *B*, we have,

$$\delta * B > A. \qquad (5.1.2)$$

First of all, with respect to $A = B_1(1) - B_1(0)$, it measures the difference between the net benefits from development and preservation in the first period. The sign of *A* may be positive, negative or zero.

Secondly, we know that

$$\begin{aligned} B &= E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi) \right] - E[B_2(1,0;\pi)] \\ &= E[\max_{d_2 \in \{0,1\}} \{B_2(0,0;\pi), B_2(0,1;\pi)] - E[B_2(1,0;\pi)]. \end{aligned} \qquad (5.1.3)$$

The sign of B, which measures the difference between the net benefits of having the flexibility to make the optimal land use decision in the second period and having no other choice but locking in the situation of development in the second period, may be positive, negative or zero.

(1) In case that $A = 0$, as long as $B > 0$, the inequality (5.1.2) will always hold. $A = 0$ implies that the DM will obtain the same payoffs in the first period no matter which strategy he/she choose in the first period. $B > 0$ implies that the DM will get more payoffs in the second period if he/she chooses preservation in the first period. Therefore, if $A = 0$ and $B > 0$, preservation will always be the best choice for the first period decision.

(2) In case that $A > 0$, in order to make the inequality (5.1.2) hold, it must be true that $B > 0$. Substitute in $\delta = \frac{1}{1+r}$, and rearrange the inequality (5.1.2), we have,

$$\frac{B}{A} - 1 > r. \tag{5.1.4}$$

$A > 0$ implies that the DM will obtain more payoffs in the first period if he/she decides to convert the land in the first period. $B > 0$ implies that the DM will get more payoffs in the second period if he/she chooses preservation in the first period. Therefore, the DM needs to make a decision between getting more payoffs now and waiting to obtain more payoffs in the future. As illustrated by the inequality (5.1.4), in order to choose preservation rather than immediate conversion in the first period, the discount rate should be lower than $\frac{B}{A} - 1$. The DM will acquire fewer payoffs in the first period by choosing preserving the land, and at the same time, he/she will get more in the second period by making this decision. We know that the future payoffs will have a higher present value when discounted by a low discount rate. And therefore, if $A > 0$, $B > 0$ and $r < \frac{B}{A} - 1$, the strategy of preservation will be favoured.

(3) In case that $A < 0$, as long as $B \geq 0$, the inequality (5.1.2) will always hold. $A < 0$ implies that the DM will obtain more payoffs in the first period if he/she preserves the land in the first period. $B \geq 0$ implies that the DM will not get fewer payoffs in the second period by choosing preservation in the first period. Therefore, if $A < 0$ and $B \geq 0$, preserving the land will always be the best choice for the first period.

(4) In case that $A < 0$, and $B < 0$, substitute in $\delta = \frac{1}{1+r}$, and rearrange the inequality (5.1.2), we have,

$$\frac{B}{A} - 1 < r. \tag{5.1.5}$$

$A < 0$ implies that the DM will obtain more payoffs in the first period if he/she decides to preserve the land in the first period. $B < 0$ implies that the DM will get more payoffs in the second period if he/she chooses development in the first period. Still, the DM needs to make a decision between getting more payoffs now and waiting to obtain more payoffs in the future. As illustrated by the inequality (5.1.5), in order to go for postponement rather than immediate conversion, the discount rate should be higher than $\frac{B}{A} - 1$. Comparing with the result obtained from inequality (5.1.4), this time the result is reversed. However, it is actually not surprising at all. The DM will acquire more payoffs in the first period by keeping the land under preservation, and at the same time, he/she will get fewer in the second period by undertaking this strategy. When the future payoffs will be discounted by a high discount rate, the future payoffs will have a lower present value. And therefore, if $A < 0$, $B < 0$ and $r > \frac{B}{A} - 1$, the strategy of preservation in the first period will be favoured.

Case 2: $\hat{V}(0) < \hat{V}(1)$ and $\hat{d}_1 = 1$. Under this situation, the land will be immediately converted in the first period, even though the DM knows that she/he will have no other choice but keep the land being developed in the second period. We have

$$B_1(0) + \delta * \left\{ E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi) \right] \right\} < B_1(1) + \delta * E[B_2(1{,}0; \pi)]. \quad (5.1.6)$$

As mentioned in case 1, $B = B_1(1) - B_1(0)$ measures the difference between the net benefits from development and preservation in the first period. $A = E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] - E[B_2(1{,}0; \pi)]$ measures the difference of the net benefits in the second period by choosing preservation and development in the first period.

Rearrange the inequality (5.1.6), and substitute in *A* and *B*, we have

$$\delta B < A. \quad (5.1.7)$$

The sign of *A* may be positive, negative or zero. And the sign of *B* may be positive, negative or zero as well.

(1) In case that $A = 0$, as long as $B < 0$, the inequality (5.1.7) will always hold. $A = 0$ implies that the DM will obtain the same amount of payoffs in the first period no matter which decision he/she chooses in the first period. $B < 0$ implies that the DM will get more payoffs in the second period if he/she chooses to develop the land in the first period. Therefore, if $A = 0$ and $B < 0$, immediate conversion will always be the best choice for the first period.

(2) In case that $A < 0$, in order to make the inequality (5.1.7) hold, it must be true that $B < 0$. Substitute in $\delta = \frac{1}{1+r}$, and rearrange the inequality (5.1.7), we have,

$$\frac{B}{A} - 1 > r. \quad (5.1.8)$$

$A < 0$ implies that the DM will get more payoffs in the first period if he/she chooses preservation in the first period. $B < 0$ implies that the DM will get more payoffs in the second period if he/she chooses development in the first period. The DM needs to make a decision between getting more payoffs now and waiting to obtain more payoffs in the future. As illustrated by the inequality (5.1.8), in order to go for immediate conversion rather than postponement, the discount rate should be lower than $\frac{B}{A} - 1$. The DM will get fewer payoffs in the first period by converting the land for development, and meanwhile, he/she will get more in the second period by undertaking this strategy. When the future payoffs will be discounted by a low discount rate, the future payoffs will have a high present value. And therefore, if $A < 0$, $B < 0$ and $r > \frac{B}{A} - 1$, the strategy of development in the first period will be favoured.

(3) In case that $A > 0$, as long as $B \leq 0$, the inequality (5.1.7) will always hold. $A > 0$ implies that the DM will obtain more payoffs in the first period if he/she converts the land in the first period. $B \leq 0$ implies that the DM will not get less payoffs in the second period by choosing development in the first period. Therefore, if $A > 0$ and $B \leq 0$, development will always be the best choice for the first period.

(4) In case that $A > 0$ and $B > 0$, substitute in $\delta = \frac{1}{1+r}$, and rearrange the inequality (5.1.7), we have,

$$\frac{B}{A} - 1 < r. \tag{5.1.9}$$

$A > 0$ implies that the DM will get more payoffs in the first period if he/she chooses development in the first period. $B > 0$ implies that the DM will get more payoffs in the second period if he/she chooses preservation in the first period. Therefore, still, the DM needs to make a decision between getting more payoffs now and waiting to obtain more payoffs in the future. As illustrated by the inequality (5.1.9), in order to go for immediate conversion rather than preservation, the discount rate should be higher than $\frac{B}{A} - 1$. Comparing with the result obtained in inequality (5.1.8), this time the result is reversed. However, it is actually not surprising at all. The DM will acquire more payoffs in the first period by choosing developing the land, and at the same time, he/she will get fewer in the second period by taking this strategy. We know that the future payoffs will have a lower present value when discounted by a higher discounted by a high discount rate. And therefore, if $A > 0$, $B > 0$ and $r > \frac{B}{A} - 1$, the decision of immediate conversion will be favoured.

<u>Case 3:</u> $\hat{V}(0) = \hat{V}(1)$ and $\hat{d}_1 = 0 \; or \; 1$. Under this situation, the DM will be indifferent between immediate conversion of land and preserving the land in the first period. We have

$$B_1(0) + \delta * \left\{E\left[\max_{d_2 \in \{0,1\}} B_2(d_1, d_2; \pi)\right]\right\} = B_1(1) + \delta * E[B_2(1; \pi)]. \tag{5.1.10}$$

Similarly, $A = B_1(1) - B_1(0)$ and $B = E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] - E[B_2(1,0; \pi)]$. The interpretations of A and B are the same as before and both of them may be positive, negative or zero.

Substitute in A and B to (5.1.10), we have

$$\delta B = A. \tag{5.1.11}$$

In order to make the equation (5.1.11) hold, it must be true that the sign of A should be the same with the sign of B. In other words, when $A = 0$, it must be true that $B = 0$; when $A > 0$, it must be the true that $B > 0$; when $A < 0$, it must be the true that $B < 0$.

In case that $A = 0$ and $B = 0$, the equation (5.1.11) will always hold. In other cases, substitute in $\delta = \frac{1}{1+r}$, and rearrange the equation (5.1.11), we have

$$\frac{B}{A} - 1 = r. \tag{5.1.12}$$

$\frac{B}{A} - 1$ is the critical discount rate under standard exponential discounting that makes the DM indifferent between conversion in the first period and postponing the developing decision to the second period.

In case that $A > 0$ and $B > 0$. When the discount rate is higher than $\frac{B}{A} - 1$, immediate conversion will be favoured in the first period. When the discount rate is lower than $\frac{B}{A} - 1$, preserving the land will be favoured in the first period.

In case that $A < 0$ and $B < 0$. When the discount rate is higher than $\frac{B}{A} - 1$, preserving the land in the first period will be favoured. When the discount rate is lower than $\frac{B}{A} - 1$, immediate conversion will be favoured in the first period.

5.1.2 Analysis under quasi-hyperbolic discounting

According to decision rules mentioned by equation (4.2.4) in the section of 4.2.2, the level of development in the first period, $\tilde{d}_1$, depends on the expected payoffs over both periods $\tilde{V}(\tilde{d}_1)$. $\tilde{d}_1$ can either be 0 or 1. Therefore, the decision about first period development will be determined by the value of $\tilde{V}(0)$ and $\tilde{V}(1)$.

Accordingly, there will be three different situations as well.

Case 1: $\tilde{V}(0) > \tilde{V}(1)$ and $\tilde{d}_1 = 0$. Under this situation, the land will not be converted in the first period, and it is optimal for the DM to wait and decide whether convert the land or not in the second period. We have,

$$B_1(0) + \beta\delta * \left\{E\left[\max_{d_2\in\{0,1\}} B_2(d_1, d_2; \pi)\right]\right\} > B_1(1) + \beta\delta * E[B_2(1; \pi)]. \qquad (5.1.13)$$

Similar with the analysis under exponential discounting, for the sake of simplification, let us define $\text{A} = B_1(1) - B_1(0)$. "$A$" is the difference between the net benefits from development and preservation in the first period. Define $B = E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right] - E[B_2(1,0; \pi)]$. "$B$" is the difference of net benefits in the second period by choosing preservation and development in the first period.

Rearrange the inequality (5.1.13), and substitute in A and B, we have,

$$\beta\delta * B > A. \qquad (5.1.14)$$

The sign of A, may be positive, negative or zero. And the sign of B may be positive, negative or zero as well.

(1) In case that $A = 0$ and $B > 0$, the inequality (5.1.14) will always hold. $A = 0$ implies that the DM will acquire the same amount of payoffs in the first period by preserving or developing the land in the first period. $B > 0$ implies that the DM will get more payoffs in the second period by choosing preserving the land in the first period. And therefore, if $A = 0$ and $B > 0$, preserving the land in the first period is always the best decision.

(2) In case that $A > 0$, in order to make the inequality (5.1.14) hold, it must be true that $B > 0$. Substitute in $\delta = \frac{1}{1+r}$, and rearrange the inequality (5.1.14), we have,

$$\frac{\beta B}{A} - 1 > r. \qquad (5.1.15)$$

$A > 0$ implies that the DM will get more payoffs in the first period if he/she chooses development in the first period. $B > 0$ implies that the DM will get more payoffs in the second period if he/she chooses preservation in the first period. Therefore, the DM needs to make a decision between getting more payoffs now and waiting to obtain more payoffs in the future. As illustrated by the inequality (5.1.15), in order to go for postponement rather than immediate conversion, the discount rate should be lower than $\frac{\beta B}{A} - 1$. The DM will obtain fewer payoffs in the first period by preserving the land, and in the meantime, he/she will get more in the second period by making this decision. We know that the future payoffs will have a higher present value when discounted by a low discount rate. And therefore, if $A > 0$, $B > 0$ and $r < \frac{\beta B}{A} - 1$, the strategy of preservation in the first period will be favoured.

(3) In case that A < 0, as long as $B \geq 0$, the inequality (5.1.14) will always hold. $A < 0$ implies that the DM will get more payoffs in the first period if he/she chooses to preserve the land in the first period. $B \geq 0$ implies that the DM will obtain more payoffs in the second period if he/she preserve the land in the first period. And therefore, if $A < 0$ and $B \geq 0$, preserving the land is always the best decision for the first period.

(4) In case that A < 0 and B < 0, substitute in $\delta = \frac{1}{1+r}$, and rearrange the inequality (5.1.14), we have,

$$\frac{\beta B}{A} - 1 < r. \tag{5.1.16}$$

$A < 0$ implies that the DM will get more payoffs in the first period if he/she chooses preservation in the first period. $B < 0$ implies that the DM will get more payoffs in the second period if he/she chooses development in the first period. Still, the DM needs to make a decision between getting more payoffs now and waiting to obtain more payoffs in the future. As illustrated by the inequality (5.1.16), in order to go for postponement rather than immediate conversion, the discount rate should be higher than $\frac{\beta B}{A} - 1$. Comparing with the result given by inequality (5.1.15), this time the result is reversed. However, it is actually not surprising at all. The DM will get more payoffs in the first period by keeping the land under preservation, and simultaneously, he/she will get fewer in the second period by undertaking this strategy. When the future payoffs will be discounted by a high discount rate, the future payoffs will have a lower present value. And therefore, if $A < 0$, $B < 0$ and $r > \frac{\beta B}{A} - 1$, the strategy of preservation in the first period will be favoured.

<u>Case 2:</u> $\tilde{V}(0) < \tilde{V}(1)$ and $\tilde{d}_1 = 1$. Under this situation, it is best for the DM to convert the land for development in the first period. We have,

$$B_1(0) + \beta\delta * \left\{E\left[\max_{d_2 \in \{0,1\}} B_2(d_1, d_2; \pi)\right]\right\} < B_1(1) + \beta\delta * E[B_2(1; \pi)]. \tag{5.1.17}$$

Similarly, $A = B_1(1) - B_1(0)$ and $B = E\left[\max_{d_2 \in \{0,1\}} B_2(d_1, d_2; \pi)\right] - E[B_2(1; \pi)]$. The meanings of A and B are as before. Rearrange the inequality (5.1.17), and substitute in A *and* B, we have,

$$\beta\delta * B < A. \tag{5.1.18}$$

The sign of A may be positive, negative or zero. And the sign of B may be positive, negative or zero.

(1) In case that $A = 0$, as long as $B < 0$, the inequality (5.1.18) will always hold. $A = 0$ implies that the DM will obtain the same amount of payoffs in the first period by choosing either preservation or development in the first period. $B < 0$ implies that the DM will get more payoffs in the second period by choosing development in the first

period. Therefore, if $A = 0$ and $B < 0$, converting the land is the best choice for the first period.

(2) In case that $A < 0$, in order to make the inequality (5.1.18) hold, it must be true that $B < 0$. Substitute in $\delta = \frac{1}{1+r}$, and rearrange the inequality (5.1.18), we have

$$\frac{\beta B}{A} - 1 > r. \tag{5.1.19}$$

$A < 0$ implies that the DM will acquire more payoffs in the first period by preserving the land in the first period. $B < 0$ implies that the DM will obtain more payoffs in the second period by choosing converting the land for development in the first period. The DM needs to make a choice between taking more payoffs now and being patient to obtain more payoffs in the future. As illustrated by the inequality (5.1.19), in order to go for immediate conversion rather than preservation, the discount rate should be lower than $\frac{\beta B}{A} - 1$. The DM will get fewer payoffs in the first period by developing the land, and in the meantime, he/she will get more in the second period by taking this strategy. We know that future payoffs will have a higher present value when discounted by a low discount rate. And therefore, if $A < 0$, $B < 0$ and $r < \frac{\beta B}{A} - 1$, the decision of immediate conversion will be favoured.

(3) In case that $A > 0$, as long as $B \leq 0$, the inequality (5.1.18) will always hold. $B \leq 0$ implies that the DM will not get fewer payoffs in the second period by choosing development in the first period. $A > 0$ implies that the DM will obtain more payoffs in the first period by choosing development in the first period. Therefore, if $A > 0$ and $B \leq 0$, development is the best decision for the first period.

(4) In case that $A > 0$ and $B > 0$, substitute in $\delta = \frac{1}{1+r}$, and rearrange the inequality (5.1.18), we have

$$\frac{\beta B}{A} - 1 < r. \tag{5.1.20}$$

$A > 0$ implies that the DM will get more payoffs in the first period if he/she chooses development in the first period. $B > 0$ implies that the DM will acquire more payoffs in the second period if he/she chooses preservation in the first period. Still, the DM needs to make a decision between getting more payoffs now and waiting to obtain more payoffs in the future. As illustrated by the inequality (5.1.20), in order to go for immediate conversion rather than preservation, the discount rate should be higher than $\frac{\beta B}{A} - 1$. The DM will obtain more payoffs in the first period by converting the land for development, and in the meantime, he/she will get less in the second period by making this decision. We know that the future payoffs will have a lower present value when discounted by a high discount rate. And therefore, if $A > 0$, $B > 0$ and $r > \frac{\beta B}{A} - 1$, the decision of immediate conversion will be favoured.

Case 3: $\tilde{V}(0) = \tilde{V}(1)$ and $\tilde{d}_1 = 0\ or 1$. Under this situation, the DM will be indifferent between immediate conversion and immediate preservation of the land. We have,

$$B_1(0) + \beta\delta * \left\{E\left[\max_{d_2\in\{0,1\}} B_2(d_1, d_2; \pi)\right]\right\} = B_1(1) + \beta\delta * E[B_2(1; \pi)]. \qquad (5.1.21)$$

Similarly, $A = B_1(1) - B_1(0)$ and $B = E\left[\max_{d_2\in\{0,1\}} B_2(d_1, d_2; \pi)\right] - E[B_2(1; \pi)]$. The interpretation of A and B is the same as before, and both of them may be positive, negative or zero.

Rearrange the inequality (5.1.21), and substitute in $A\ and\ B,$ we have,

$$\beta\delta * B = A. \qquad (5.1.22)$$

In order to make this equation hold, it must be true that the sign of A and the sign of B should be the same. In other words, when $A = 0$, it must be true that $B = 0$. When $A > 0$, it must be true that $B > 0$. when $A < 0$, it must be true that $B < 0$.

In case that $A = 0$ and $B = 0$, the equation (5.1.22) will always hold. In other cases, substitute in $= \frac{1}{1+r}$, and rearrange equation (5.1.22), we have

$$\frac{\beta B}{A} - 1 = r. \qquad (5.1.23)$$

$\frac{\beta B}{A} - 1$ is the critical discount rate under hyperbolic discounting that makes the DM indifferent between conversion in the first period and postponing the developing decision to the second period.

In case that $A > 0$ and $B > 0$. When the discount rate is higher than $\frac{\beta B}{A} - 1$, immediate conversion will be favoured, and when the discount rate is lower than $\frac{\beta B}{A} - 1$, postponement will be favoured.

In case that $A < 0$ and $B < 0$. When the discount rate is higher than $\frac{\beta B}{A} - 1$, postponement will be favoured, and when the discount rate is lower than $\frac{\beta B}{A} - 1$, immediate conversion will be favoured.

5.2 Discussion

5.2.1 A change in the value of “A” and “B”

As illustrated in section 5.1, $\mathrm{A} = B_1(1) - B_1(0)$, $B = E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right] - E[B_2(1{,}0; \pi)]$. “$A$” measures the difference between net benefits from development and preservation in the first period. “B” measures the difference of net benefits in the second period by choosing preservation and development in the first period.

Therefore, under exponential discounting,

$$\begin{aligned} &\hat{V}(0) - \hat{V}(1) \\ &= \mathrm{B}_1(0) - \mathrm{B}_1(1) + \delta * \left\{\mathrm{E}\left[\max_{\mathrm{d}_2\in\{0,1\}} \mathrm{B}_2(0, \mathrm{d}_2; \pi)\right] - \mathrm{E}[\mathrm{B}_2(1{,}0; \pi)]\right\} \\ &= \delta B - A. \end{aligned} \tag{5.2.1}$$

Under hyperbolic discounting,

$$\begin{aligned} &\tilde{V}(0) - \tilde{V}(1) \\ &= \mathrm{B}_1(0) - \mathrm{B}_1(1) + \beta\delta * \left\{\mathrm{E}\left[\max_{\mathrm{d}_2\in\{0,1\}} \mathrm{B}_2(0, \mathrm{d}_2; \pi)\right] - \mathrm{E}[\mathrm{B}_2(1{,}0; \pi)]\right\} \\ &= \beta\delta B - A. \end{aligned} \tag{5.2.2}$$

(1) Ceteris paribus, when “A” increases, both $\hat{V}(0) - \hat{V}(1)$ and $\tilde{V}(0) - \tilde{V}(1)$ will decrease. Hence, an increase in “A” will favour the decision of immediate converting the land for development purpose in the first period. In contrast, when “A” decreases, both $\hat{V}(0) - \hat{V}(1)$ and $\tilde{V}(0) - \tilde{V}(1)$ will increase. Therefore, a decrease in “A” will favour the decision of preserving the land in the first period. The economic insight behind this is not that complex. Keeping all the other factors unchanged, an increase in “A” implies an increase of the net benefit from development or a decrease of the net benefit from preservation. Accordingly, the DM will be motivated to grab the extra payoffs by developing the land. Similarly, with all the other factors being constant, a decrease in “A” implies a decline of net benefit from development or a rise of net benefit from preservation. As a result, preserving the land will be preferred in the first period.

(2) Ceteris paribus, when “B” increases, both $\hat{V}(0) - \hat{V}(1)$ and $\tilde{V}(0) - \tilde{V}(1)$ will increase. Thus, an increase in “B” will favour the decision of preserving the land in the first period. In contrast, when “B” decreases, both $\hat{V}(0) - \hat{V}(1)$ and $\tilde{V}(0) - \tilde{V}(1)$ will decrease. Therefore, a decrease in “B” will favour the decision of immediate converting the land for development purpose in the first period. The economic insight behind this is not that complex either. Holding other factors constant, an increase in “B” implies an increase of the net benefit from preservation of second period by choosing preserving the land in the first period or a decline of the net benefit from development of second period

by choosing developing the land in the first period. Consequently, the DM will be motivated to wait and choose the preservation in the first period. Similarly, with other factors being unchanged, a decline in "B" implies a decrease of the net benefit from preservation of second period by choosing preserving the land in the first period or a rise of the net benefit from development of second period by choosing developing the land in the first period. As a result, the DM will tend to convert the land in the first period.

5.2.2 A change in the discount rate

According to the analysis in section 5.1, we know that,

(1) If $A = 0$ and at the same time $B > 0$ or $A < 0$ and at the same time $B \geq 0$, preserving the land in the first period in order to have the option to make flexible decisions in the second period will always be the best choice for the DM. In this case, the change of discount rate does not have any impact on the development decision in the first period.

(2) If $A = 0$ and meanwhile $B < 0$ or $A > 0$ and meanwhile $B \leq 0$, immediate converting the land for development purpose in the first period will always be the best decision for the DM. Similarly, in this situation, the change of discount rate does not influence the development decision in the first period.

(3) In case that $A > 0$ and $B > 0$, with the increase of discount rate, the overall payoff $\hat{V}(0)$ and $\tilde{V}(0)$ will be reduced more than the overall payoff $\hat{V}(1)$ and $\tilde{V}(1)$. Therefore, an increase of discount rate will favour the decision of immediate converting the land. Similarly, with the decrease of discount rate, the overall payoff $\hat{V}(0)$ and $\tilde{V}(0)$ will be increased more than the overall payoff $\hat{V}(1)$ and $\tilde{V}(1)$. Thus, a decrease of the discount rate will favour the decision of preserving the land in the first period.

    Proof. See appendix 1.

(4) In case that $A < 0$ and $B < 0$, with the increase of discount rate, the overall payoff $\hat{V}(1)$ and $\tilde{V}(1)$ will be reduced more than the overall payoff $\hat{V}(0)$ and $\tilde{V}(0)$. Therefore, an increase of discount rate will favour the decision of preserving the land in the first period. Similarly, with the decrease of discount rate, the overall payoff $\hat{V}(1)$ and $\tilde{V}(1)$ will be increased more than the overall payoff $\hat{V}(0)$ and $\tilde{V}(0)$. Therefore, a decrease of the discount rate will favour the decision of immediate conversion of the land.

    Proof. See appendix 2.

The changes in discount rate and the corresponding influences for the first period land allocation decisions are summarized in table 1.

Table 1. Changes in discount rate and the corresponding first period land allocation decisions

First period land allocation decision	Exponential discounting	Quasi-hyperbolic discounting
Preserving the land	$A = 0, B > 0$ $A < 0, B \geq 0$ $A > 0, B > 0\ and\ r < \frac{B}{A} - 1$ $A < 0, B < 0\ and\ r > \frac{B}{A} - 1$	$A = 0, B > 0$ $A < 0, B \geq 0$ $A > 0, B > 0\ and\ r < \frac{\beta B}{A} - 1$ $A < 0, B < 0\ and\ r > \frac{\beta B}{A} - 1$
Developing the land	$A = 0, B < 0$ $A > 0, B \leq 0$ $A > 0, B > 0\ and\ r > \frac{B}{A} - 1$ $A < 0, B < 0\ and\ r < \frac{B}{A} - 1$	$A = 0, B < 0$ $A > 0, B \leq 0$ $A > 0, B > 0\ and\ r > \frac{\beta B}{A} - 1$ $A < 0, B < 0\ and\ r < \frac{\beta B}{A} - 1$
Being indifferent between preservation and development	$A = 0, B = 0$ $A > 0, B > 0\ and\ r = \frac{B}{A} - 1$ $A < 0, B < 0\ and\ r = \frac{B}{A} - 1$	$A = 0, B = 0$ $A > 0, B > 0\ and\ r = \frac{\beta B}{A} - 1$ $A < 0, B < 0\ and\ r = \frac{\beta B}{A} - 1$

Besides, the critical discount rate, r^*_{exp}, under exponential discounting, which makes the DM indifferent between immediate development and postponement of the development, is $\frac{B}{A} - 1$. And the critical discount rate, r^*_{hyp}, under quasi-hyperbolic discounting, which makes the DM indifferent between immediate development and postponement of the development is $\frac{\beta B}{A} - 1$. Apparently, we have $r^*_{hyp} < r^*_{exp}$. That is, the required critical discount rate for hyperbolic discounting is lower than exponential discounting.

(1) In case that $A > 0$ and $B > 0$. When the discount rate $r < r^*_{hyp}$, under both discounting functions, the DM will choose to preserve the land. When the discount rate $r > r^*_{exp}$, under both discounting functions, the DM will choose to convert the land. The interesting thing is, when the discount rate $r \in (r^*_{hyp}, r^*_{exp})$, under the standard exponential discounting, the DM will decide to preserve the land in the first period; while on the contrary, under hyperbolic discounting, the DM will decide to convert the land in the first period. Therefore, ceteris paribus, when there is an immediate reward by converting the land for developing purpose, under hyperbolic discounting, the DM tends to be more impatient and convert the land in the first period.

(2) In case that $A < 0$ and $B < 0$. When the discount rate $r < r^*_{hyp}$, under both discounting functions, the DM will choose to immediate converting the land for development purposes. When the discount rate $r > r^*_{exp}$, under both discounting functions, the DM will choose to preserve the land in the first period. There will be a certain range, that is, when the discount rate $r \in (r^*_{hyp}, r^*_{exp})$, under the standard exponential discounting, the DM will decide to immediate convert the land; in contrast, under hyperbolic discounting, the DM will choose to preserve the land in the first period. Therefore, ceteris paribus, when there is an immediate cost by converting the land for developing purpose, under hyperbolic discounting, the DM tends to wait and decide later.

The comparison between the critical discounting rates under exponential discounting and quasi-hyperbolic discounting are summarized in table 2.

Table 2. First period land allocation decision under different discounting schemes

	Discounting schemes	$r < r^*_{hyp}$	$r \in (r^*_{hyp}, r^*_{exp})$	$r > r^*_{exp}$
$A > 0$ *and* $B > 0$	Exponential discounting	Preservation	Preservation	Development
	Quasi-hyperbolic discounting	Preservation	Development	Development
$A < 0$ *and* $B < 0$	Exponential discounting	Development	Development	Preservation
	Quasi-hyperbolic discounting	Development	Preservation	Preservation

5.2.3 A change in the probability

In order to illustrate the influence of probability for the land allocation decisions, we consider the following situation: suppose there are only two possible states in the second period, state 1 and state 2. And the DM holds the belief that, state 1 will occur with probability π, and state 2 will occur with probability $1 - \pi$. When state 1 happens, net benefit from second period development, $B_2^1(0,1)$, is higher than the net benefit from second period preservation, i.e. $B_2^1(0,1) \geq B_2^1(0,0)$; when state 2 occurs, net benefit from development, $B_2^1(1,0)$, is lower than the net benefit form preservation, i.e. $B_2^2(1,0) < B_2^2(0,0)$.

Therefore, the overall payoffs under hyperbolic discounting:

In case that $\tilde{d}_1 = 0$,

$$\tilde{V}(0) = B_1(0) + \beta\delta * \left\{E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right]\right\} \quad (5.2.3)$$

$$= \mathrm{B}_1(0) + \beta\delta * \left\{ E\left[\max_{d_2 \in \{0,1\}} \{B_2(0{,}1;\pi), B_2(0{,}0;\pi)\} \right] \right\}$$
$$= B_1(0) + \beta\delta * [\pi B_2^1(0{,}1) + (1-\pi) B_2^2(0{,}0)].$$

In case that $\tilde{d}_1 = 1$,

$$\tilde{V}(1) = B_1(1) + \beta\delta * E[B_2(1{,}0;\pi)]$$
$$= \mathrm{B}_1(1) + \beta\delta * [\pi B_2^1(1{,}0) + (1-\pi) B_2^2(1{,}0)] \tag{5.2.4}$$

Therefore, when the probability of state 1 occurs is quite high, i.e. $\pi \to 1$, we have in the second period, $B_2^1(0{,}1) \geq B_2(0{,}0)$. And therefore,

$$\lim_{\pi \to 1} \tilde{V}(0) = B_1(0) + \beta\delta B_2^1(0{,}1) \tag{5.2.5}$$

$$\lim_{\pi \to 1} \tilde{V}(1) = B_1(1) + \beta\delta B_2^1(1{,}0) \tag{5.2.6}$$

From (5.2.5) and (5.2.6), we can see that, the difference between $\lim_{\pi\to 1} \tilde{V}(0)$ and $\lim_{\pi\to 1} \tilde{V}(1)$ depends on the difference between $B_1(0)$ and $B_1(1)$, difference between $B_2^1(0{,}1)$ and $B_2^1(1{,}0)$, the quasi-hyperbolic parameter β and the exponential discounting factor δ.

By assumption, we have $B_2^1(0{,}1) < B_2^1(1{,}0)$. Then,

$$\lim_{\pi \to 1} B = \lim_{\pi \to 1} E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi) \right] - \lim_{\pi \to 1} E[B_2(1{,}0;\pi)]$$
$$= B_2^1(0{,}1) - B_2^1(1{,}0) < 0. \tag{5.2.7}$$

If $A = B_1(1) - B_1(0) \geq 0$, according to the analysis in section 5.2.2, the decision of development in the first period will always be favoured. In other words, immediate conversion is the best choice under both discounting schemes.

If $A < 0$, under hyperbolic discounting, future net benefits are discounted more than under exponential discounting. According to the analysis in section 5.2.2, preserving the land in the first period will be favoured under hyperbolic discounting.

When the probability of state 2 occurs is quite high, i.e. $\pi \to 0$, we have in the second period, $B_2^2(1{,}0) < B_2^2(0{,}0)$. And thus,

$$\lim_{\pi \to 0} \tilde{V}(0) = B_1(0) + \beta\delta B_2^2(0{,}0) \tag{5.2.8}$$

$$\lim_{\pi \to 0} \tilde{V}(1) = B_1(1) + \beta\delta B_2^2(1{,}0) \tag{5.2.9}$$

From (5.2.8) and (5.2.9), we can see that, the difference between $\lim_{\pi\to 0} \tilde{V}(0)$ and $\lim_{\pi\to 0} \tilde{V}(1)$ depends on difference between $B_1(0)$ and $B_1(1)$, difference between

$B_2^2(0,0)$ and $B_2^2(1,0)$, the quasi-hyperbolic parameter β and the exponential discounting factor δ.

By assumption, we have $B_2^2(0,0) > B_2^2(1,0)$. Then,

$$\lim_{\pi\to 0} B = \lim_{\pi\to 0} E\left[\max_{d_2\in\{0,1\}} B_2(0,d_2;\pi)\right] - \lim_{\pi\to 1} E[B_2(1,0;\pi)] \\ = B_2^2(0,0) - B_2^2(1,0) > 0. \quad (5.2.10)$$

If $A = B_1(1) - B_1(0) \leq 0$, according to the analysis in section 5.2.2, the decision of preservation in the first period will always be favoured. In other words, preserving the land in the first period is the best choice under both discounting schemes.

If $A > 0$, under hyperbolic discounting, future net benefits are discounted more than under exponential discounting. According to the analysis in section 5.2.2, converting the land for development purpose in the first period will be favoured under hyperbolic discounting.

5.2.4 The impact on option value

The analysis above focuses on the impact of different discounting schemes about the first period land allocation decision under the hypothesis of being able to take advantage of future information. In order to illustrate influences of exponential discounting and hyperbolic discounting on the quasi-option value, discount factor will be introduced to the two-period framework set out by Mäler and Fisher (2005). Two scenarios, one scenario with forthcoming complete information resolving the uncertainty and the other scenario without this expectation, will be presented.

<u>Scenario 1</u>: with prospect of available information to resolve the uncertainty, the DM can use this information to make the best decision in the second period given any level of development in the first period. The overall payoffs to the DM is denoted by $\hat{F}(d_1)$, and the discount factor, either hyperbolic discounting or exponential discounting, is denoted as DF.

If $d_1 = 0$,

$$\hat{F}(0) = B_1(0) + \hat{B}_2(d_2) * DF = B_1(0) + E\left[\max_{d_2\in\{0,1\}} B_2(0,d_2;\pi)\right] * DF \quad (5.2.11)$$

If $d_1 = 1$,

$$\hat{F}(1) = B_1(1) + \hat{B}_2(0) * DF = B_1(1) + E[B_2(1,0;\pi)] * DF. \quad (5.2.12)$$

Decision rules under scenario1:

$$\max_{d_1} \hat{F}(d_1) = \max\{\hat{F}(0), \hat{F}(1)\}. \tag{5.2.13}$$

Scenario 2: No expectation of information that would permit DM to resolve the uncertainty and the DM has to make a decision given any level of development in the first period. The overall payoff to the DM is denoted by $\tilde{F}(d_1)$.

If $d_1 = 0$,

$$\tilde{F}(0) = B_1(0) + \tilde{B}_2(d_2) * DF = B_1(0) + \max_{d_2 \in \{0,1\}} E[B_2(0, d_2; \pi)] * DF \tag{5.2.14}$$

If $d_1 = 1$,

$$\tilde{F}(1) = B_1(1) + \tilde{B}_2(0) * DF = B_1(1) + E[B_2(1,0; \pi)] * DF. \tag{5.2.15}$$

Decision rules under scenario 2:

$$\max_{d_1} \tilde{F}(d_1) = \max\{\tilde{F}(0), \tilde{F}(1)\}. \tag{5.2.16}$$

We know that, by preserving the land in the first period,

$$\begin{aligned} &\hat{F}(0) - \tilde{F}(0) \\ &= \{B_1(0) + E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] * DF\} - \\ &\quad \{B_1(0) + \max_{d_2 \in \{0,1\}} E[B_2(0, d_2; \pi)] * DF\} \\ &= DF * \left\{E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] - \max_{d_2 \in \{0,1\}} E[B_2(0, d_2; \pi)]\right\}. \end{aligned} \tag{5.2.17}$$

According to Jensen's Inequality (Mäler and Fisher, 2005), we know that, the expected value of a convex function of a random variable is no less than the convex function of the expected value of the random variable. And the maximum function is a convex function[2]. We have

$$E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] - \max_{d_2 \in \{0,1\}} E[B_2(0, d_2; \pi)] \geq 0. \tag{5.2.18}$$

As a result, it will always be true that,

$$\begin{aligned} &\hat{F}(0) - \tilde{F}(0) = \\ &DF * \left\{E\left[\max_{d_2 \in \{0,1\}} B_2(0, d_2; \pi)\right] - \max_{d_2 \in \{0,1\}} E[B_2(0, d_2; \pi)]\right\} \geq 0 \end{aligned} \tag{5.2.19}$$

[2] For details, see Appendix 3.

$DF \in [0,1]$, is the discount factor. And what is interesting is that, no matter exponential discounting, where $DF = \delta$, or hyperbolic discounting, where $DF = \beta\delta$, we always have a nonnegative quasi-option value. This nonnegative option value exists if and only if the best strategy in the first period is preserving the land.

Clearly, the hyperbolic discounting factor $\beta\delta$ is lower than the exponential discounting factor δ. And thus, under hyperbolic discounting, the quasi-option value will be no higher than under exponential discounting.

Under both scenarios, we will have $\hat{F}(1) = \tilde{F}(1)$. In case that $\hat{F}(1) > \hat{F}(0)$ and $\tilde{F}(1) > \tilde{F}(0)$, converting the land for developing purpose in the first period will be the best choice under both scenarios. Here, we consider the interesting case, where

$$\hat{F}(0) \geq \hat{F}(1) = \tilde{F}(1) \geq \tilde{F}(0). \quad (5.2.20)$$

This implication for the situation is that there will be cases that development is the optimal strategy in the first period with the prospect of no information; however, preservation is the optimal strategy in the first period with the prospect of information. In other words, current development is less likely with the prospect of forthcoming information to resolve all the uncertainty.

6 Conclusions and recommendations

6.1 Synthesis of the results

This thesis targets to model the best land allocation decision between two alternatives: preservation and development. The factors affecting decision rules under a more appropriate discounting scheme, hyperbolic discounting is taken into account in this study. In addition, decision rules obtained under exponential discounting are discussed for comparison.

In this two-period model, it has been shown that, in case of immediate reward from converting the land for development purpose, the land will be converted earlier under hyperbolic discounting. On the contrary, in case of immediate cost from converting the land for development purpose, the decision about land conversion will be postponed under hyperbolic discounting.

In addition, when an optimistic state associated with development occurs, or a high probability is attached to this optimistic state, the difference of net benefits in the second period by choosing preservation and development in the first period will be negative. Land will be converted later under hyperbolic discounting. In contrast, when a pessimistic state associated with development occurs, or a high probability is attached to this pessimistic state, the difference of net benefits in the second period by choosing preservation and development in the first period will be positive. The land will be converted earlier under hyperbolic discounting.

This study has also shown that, a quasi-option value exists if and only if preserving is the best strategy for the first period. This quasi-option value will be reduced more under hyperbolic discounting than under exponential discounting.

6.2 Recommendations

This study has shown that, when an immediate reward is associated with the land development, the land will be converted earlier under hyperbolic discounting than under exponential discounting. As a result, this could lead to the over-development of land. In order to avoid the overdevelopment of land, policy targets to compensate the landowner for the forgone benefit from development should be considered.

On the contrary, when an immediate cost is associated with the land development, the land will be converted later under hyperbolic discounting than under exponential discounting. As a result, this could potentially result in over-preservation of land. In order to keep a certain amount of land being developed, policy should compensate the landowner for the forgone benefit from preservation.

6.3 Suggestions for future research

This study has shown the impact on option value and the optimal land allocation decision by introducing different discounting functions, i.e. exponential discounting and hyperbolic discounting. These results have been illustrated by a simple but effective two-period framework.

In this analysis, the main difference between hyperbolic discounting and exponential discounting is that future payoffs are discounted more under hyperbolic discounting and discounted less under exponential discounting. Or equivalently, the decision rules under hyperbolic discounting could be obtained under exponential discounting by using a high discount rate. Indeed, it holds within a two-period framework. However, it will definitely not be the case for a multi-period analysis, for which, at least three periods should be taken into account.

The fundamental difference between exponential discounting and hyperbolic discounting is that exponential discounting is time consistent and hyperbolic discounting will lead to time inconsistent behaviour. Time consistency means that individual's preferences toward future payoffs are the same no matter when he/she is asked. On the contrary, time inconsistency implies that individual's preferences toward future payoffs will change according to how delayed the future payoffs are from the time for making the decision. A large amount of observations, psychological and experimental studies have shown that individuals' preferences over benefits and costs at different points in time are actually time-inconsistent. And therefore, individual's intertemporal choices are better illustrated by hyperbolic discounting rather than exponential discounting.

A common strategy used to model time-inconsistent preference is: Individual at each time point is modelled as a separate "*self*". The current "*self*" is able to behave optimally to maximize current self's utility; while future "*selves*" are assumed to behave optimally in future periods to maximize future selves' utility. As mentioned by Pollak (1968), there are two extreme assumptions about an individual's belief about his/her future "*selves*": A person could be naïve, that is, the person did not realize the problem of preference reversal. In other words, with the approaching of time, future "*selves*" could make another decision, for which the current "*self*" would not prefer today. A person could be sophisticated, that is, the person took into account the preference reversal of future selves. Further research by O'Donoghue and Rabin (2001) suggested that individual could also be partially naïve. Partial naivety implies the person realized he/she might have a problem of future preference reversal, but the magnitude was underestimated by the current "*self*".

In this study, the effect of time-inconsistency for hyperbolic discounting has not been illustrated. For future research about optimal land allocation decisions, an extension of this two period framework, a three-period model may be considered to describe the effect of time inconsistency under hyperbolic discounting.

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Appendix 1

We know that

$$\hat{V}(0) = B_1(0) + E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right] * \frac{1}{1+r},$$

$$and\ \hat{V}(1) = B_1(1) + E[B_2(1; 0; \pi)] * \frac{1}{1+r}.$$

Therefore,

$$\frac{d\hat{V}(0)}{dr} = -\frac{1}{(1+r)^2}\left\{E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right]\right\},$$

$$and\ \frac{d\hat{V}(1)}{dr} = -\frac{1}{(1+r)^2}\{E[B_2(1,0; \pi)\}.$$

Clearly, when $B > 0$, that is, $E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right] > E[B_2(1,0; \pi)]$, we have, $\frac{d\hat{V}(0)}{dr} < \frac{d\hat{V}(1)}{dr} < 0$

Therefore, when r increase, $\hat{V}(0)$ will decrease more than the decrease of $\hat{V}(1)$; when r decreases, $\hat{V}(0)$ will increase more than the increase of $\hat{V}(1)$.

Similarly,

$$\frac{d\tilde{V}(0)}{dr} = -\frac{1}{(1+r)^2}\beta\left\{E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right]\right\},$$

$$and\ \frac{d\tilde{V}(1)}{dr} = -\frac{1}{(1+r)^2}\beta\{E[B_2(1,0; \pi)\}.$$

When $B > 0$, that is, $E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right] > E[B_2(1,0; \pi)]$, we have $\frac{d\tilde{V}(0)}{dr} < \frac{d\tilde{V}(1)}{dr} < 0$.

Therefore, when r increase, $\tilde{V}(0)$ will decrease more than the decrease of $\tilde{V}(1)$; when r decreases, $\tilde{V}(0)$ will increase more than the increase of $\tilde{V}(1)$ as well.

Appendix 2

We know that

$$\hat{V}(0) = B_1(0) + E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right] * \frac{1}{1+r},$$

$$and\ \hat{V}(1) = B_1(1) + E[B_2(1; 0; \pi)] * \frac{1}{1+r}.$$

Therefore,

$$\frac{d\hat{V}(0)}{dr} = -\frac{1}{(1+r)^2}\left\{E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right]\right\},$$

$$and\ \frac{d\hat{V}(1)}{dr} = -\frac{1}{(1+r)^2}\{E[B_2(1,0; \pi)]\}.$$

Clearly, when B< 0, that is, $E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right] < E[B_2(1,0; \pi)$, we have, $\frac{d\hat{V}(1)}{dr} < \frac{d\hat{V}(0)}{dr} < 0$.

Therefore, when r increases, $\hat{V}(1)$ will decrease more than the decrease of $\hat{V}(0)$; when r decreases, $\hat{V}(1)$ will increase more than the increase of $\hat{V}(0)$.

Similarly,

$$\frac{d\tilde{V}(0)}{dr} = -\frac{1}{(1+r)^2}\beta\left\{E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right]\right\},$$

$$\text{and}\ \frac{d\tilde{V}(1)}{dr} = -\frac{1}{(1+r)^2}\beta\{E[B_2(1,0; \pi)]\}.$$

When $B < 0$, that is, $E\left[\max_{d_2\in\{0,1\}} B_2(0, d_2; \pi)\right] < E[B_2(1,0; \pi)]$, we have $\frac{d\tilde{V}(1)}{dr} < \frac{d\tilde{V}(0)}{dr} < 0$.

Therefore, when r increases, $\tilde{V}(1)$ will decrease more than the decrease of $\tilde{V}(0)$; when r decreases, $\tilde{V}(1)$ will increase more than the increase of $\tilde{V}(0)$.

Appendix 3

According to Jensen's Inequality (Mäler and Fisher, 2005), we know that, the expected value of a convex function of a random variable is no less than the convex function of the expected value of the random variable.

The maximum function is a convex function. Therefore,

$$E\left[\max_{d_2\in\{0,1\}} B_2(0,d_2;\pi)\right] - \max_{d_2\in\{0,1\}} E[B_2(0,d_2;\pi)] \geq 0.$$